

Supercurrent in p -wave Holographic Superconductor

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Abstract

The p -wave and $p + ip$ -wave holographic superconductors with fixed DC supercurrent are studied by introducing a non-vanishing vector potential. We find that close to the critical temperature T_c of zero current, the numerical results of both the p wave model and the $p + ip$ model are the same as those of Ginzburg-Landau (G-L) theory, for example, the critical current $j_c \sim (T_c - T)^{3/2}$ and the phase transition in the presence of a DC current is a first order transition. Besides the similar results between both models, the $p + ip$ superconductor shows isotropic behavior for the supercurrent, while the p -wave superconductor shows anisotropic behavior for the supercurrent.

I. INTRODUCTION

The correspondence between gravity theory and quantum field theory (gravity/gauge duality) [1–4] has provided a novel approach to study the strongly coupled systems in condensed matter physics. This duality is a weak/strong duality, which means that we can study a strongly coupled field theory on the boundary by studying a weakly coupled quantum gravity theory on the bulk. Furthermore, in the large N limit we need only classical gravity in the bulk. One important application of AdS/CM is the holographic superconductors. The holographic superconductors can be realized because the fact that an asymptotic AdS black hole coupled with matter fields will have hair below a critical temperature and consequently it will be superconducting [5]. While above the critical temperature, the black hole has no hair, which corresponds to the normal state.

There are several holographic models of superconductor with different matter sectors. The model of s -wave holographic superconductor contains a black hole, a charged scalar field coupled to a Maxwell field[6–10]. In Ref. [11], the scalar field condensation instability of rotating anti-de Sitter black holes was also studied. If the charged scalar field in the s -wave model is replaced by a charged symmetric traceless tensor field, we will get a d -wave holographic superconductor [12–16]. The phases of an s -wave holographic superconductor with fixed superfluidity velocity were studied in Refs. [17–19], while the phases with fixed DC current was studied in Ref. [18, 20]. It is found that the s -wave holographic superconductor with DC current shows the same results as the G-L theory for superconducting films, for example, the phase transition at T_c in the presence of a DC current becomes a first order transition, the critical current $j_c \sim (T_c - T)^{3/2}$ close to T_c .

The p -wave holographic superconductor composed of non-Abelian gauge fields (the matter sector) and a black hole background (the gravity sector) was first studied in Ref. [21]. The appearance of superconductivity is due to the condensate of non-Abelian gauge fields in the theory. The properties of this non-Abelian holographic

superconductor has been studied in Refs. [22–34] (for a review one can see Ref. [35]). The model with fixed superfluidity velocity has also been studied in Ref. [27]. It is found that the phase transition at small velocity is a second order transition while at large velocity it is a first order transition. This is similar to the s -wave model with fixed velocity studied in Refs. [17–19]. With the familiarity between these two holographic superconductor models with fixed velocity, we expect that the results similar to those of an s -wave model with DC currents can also appear in the p -wave model. Since the study of p -wave holographic superconductor with fixed DC current is lacking, we study this issue in this paper. The results we *obtain* are: (a) At any fixed DC current the superconducting phase transition is a first order transition. (b) At a fixed temperature close to T_c , the j_y versus v_y curve has exactly the same features as that of the G-L theory. (c) The critical current $j_c \sim (T_c - T)^{3/2}$ near T_c . (d) Near T_c , the squared ratio of the maximal condensate to the minimal condensate is equal to two thirds at a fixed temperature; the maximal value of condensation corresponds to the zero current while the minimal value of condensation corresponds to the critical current. These results are also similar to those of the s -wave model [18, 20]. In this paper we also study the $p + ip$ wave holographic superconductor with DC current, which shows results similar to the p -wave one. However, the supercurrent in the $p + ip$ wave model is isotropic, and the p -wave superconductor shows anisotropic behavior for the supercurrent. The anisotropic behavior of the supercurrent in the p -wave superconductor also differentiates the p -wave superconductor from the s -wave superconductor.

The organization of this paper is as follows. In section II, we first review the dual gravity theory of the p -wave superconductor with DC current along the y direction and the equations of motion are also given. Then by numerically solving these equations, we obtain our results, such as the relation between the condensation and the temperature and the phase diagram. A comparison of these results to that of G-L theory and the s -wave model is also presented. In section III, we give some results

for the $p + ip$ -wave superconductors with DC current along the y direction, which are similar to that of the p -wave model. The study of the p wave and $p + ip$ wave holographic superconductors with a DC current j_x in the x direction is given in section IV. Finally, the discussion and conclusions are presented in section V.

II. p -WAVE HOLOGRAPHIC SUPERCONDUCTOR WITH CURRENT

In this section, we first review the action of the non-Abelian holographic superconductor and give the equations of motion (EOMs) of the model with a nonvanishing vector potential. Then, after numerically solving the EOMs, we give the results and their physical meanings.

A. The Dual Gravity Theory

The action of the p -wave holographic superconductor includes the Einstein-Hilbert action and an $SU(2)$ gauge field, which is called the Einstein-Yang-Mills (EYM) theory. It has the following action [5]

$$S_{\text{EYM}} = \int \sqrt{-g} d^4x \left[\frac{1}{2\kappa_4^2} \left(R + \frac{6}{L^2} \right) - \frac{L^2}{2g_{\text{YM}}^2} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) \right], \quad (\text{II.1})$$

where g_{YM} is the gauge coupling constant and $F_{\mu\nu} = T^a F_{\mu\nu}^a = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$ is the field strength of the gauge field $A = A_\mu dx^\mu = T^a A_\mu^a dx^\mu$. For the $SU(2)$ gauge symmetry, $[T^a, T^b] = i\epsilon^{abc} T^c$ and $\text{Tr}(T^a T^b) = \delta^{ab}/2$, where ϵ^{abc} is the totally anti-symmetric tensor with $\epsilon^{123} = 1$. The Yang-Mills Lagrangian becomes $\text{Tr}(F_{\mu\nu} F^{\mu\nu}) = F_{\mu\nu}^a F^{a\mu\nu}/2$ with the field strength components $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + \epsilon^{abc} A_\mu^b A_\nu^c$.

Working in the probe limit in which the matter fields do not backreact on the metric as in Refs. [21–23] and taking the planar Schwarzschild-AdS ansatz, the black hole metric reads (we use mostly plus signature for the metric)

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{L^2}(dx^2 + dy^2), \quad (\text{II.2})$$

where the metric function $f(r)$ is

$$f(r) = \frac{r^2}{L^2} \left(1 - \frac{r_0^3}{r^3}\right). \quad (\text{II.3})$$

L and r_0 are the radius of the AdS spacetime and the horizon radius of the black hole, respectively. We can set $L = 1$. Then the Hawking temperature of the black hole reads

$$T = \frac{3r_0}{4\pi}, \quad (\text{II.4})$$

which is also the temperature of the dual gauge theory living on the boundary of the AdS spacetime.

It is convenient to introduce a new coordinate $z = 1/r$. The metric (II.2) then becomes

$$ds^2 = \frac{1}{z^2} \left(-h(z)dt^2 + dx^2 + dy^2 + \frac{dz^2}{h(z)} \right), \quad (\text{II.5})$$

where $h(z) = 1 - (z/z_h)^3$ and $z_h = 1/r_0$ is the horizon. For convenience we set $z_h = 1$ in our calculation.

Using the Euler-Lagrange equations, one can obtain the equations of motion for the gauge fields,

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} F^{a\mu\nu}) + \epsilon^{abc} A_\mu^b F^{c\mu\nu} = 0. \quad (\text{II.6})$$

For the p -wave backgrounds, in order to study the DC current of the model, we need a non-vanishing vector potential. Then the ansatz takes the following form,

$$A = \phi(z)T^3dt + A_y^3(z)T^3dy + w(z)T^1dx. \quad (\text{II.7})$$

Here the $U(1)$ subgroup of $SU(2)$ generated by T^3 is identified to the electromagnetic gauge group [21] and ϕ is the electrostatic potential, which must vanish at the horizon for the gauge field in order for ϕdt to be well-defined, but need not vanish at infinity. Thus the black hole can carry charge through the condensation of w , which spontaneously breaks the $U(1)$ gauge symmetry. This is a Higgs mechanism. With this ansatz (II.7), we can derive the equations of motion,

$$(-1+z^3)^2 \frac{d^2 w(z)}{dz^2} + 3z^2(-1+z^3) \frac{dw(z)}{dz} + \phi^2(z)w(z) + (-1+z^3)(A_y^3(z))^2 w(z) = 0, \quad (\text{II.8})$$

$$(-1 + z^3) \frac{d^2 \phi''(z)}{dz^2} + \phi(z) w^2(z) = 0, \quad (\text{II.9})$$

and

$$(-1 + z^3) \frac{d^2 A_y^3(z)}{dz^2} + 3z^2 \frac{dA_y^3(z)}{dz} + A_y^3(z) w^2(z) = 0. \quad (\text{II.10})$$

In order to solve these equations we need to specify the boundary conditions on both the boundary and the horizon. On the horizon $z = z_h$, the scalar potential ϕ should vanish at the horizon in order to make ϕdt well defined. At the horizon the fields w and A_y^3 should be regular. On the boundary, the asymptotic behaviors of the three fields take the following form

$$w = \frac{\langle \mathcal{O} \rangle}{\sqrt{2}} z + \dots, \quad (\text{II.11})$$

$$\phi = \mu - \rho z + \dots, \quad (\text{II.12})$$

$$A_y^3 = v_y - j_y z + \dots. \quad (\text{II.13})$$

From the AdS/CFT dictionary we explain μ as the chemical potential, ρ as the charge density, v_y the superfluid velocity and j_y the supercurrent along the y direction of the boundary field theory. $\langle \mathcal{O} \rangle$ is the order parameter of the superconducting phase. The constant term in Eq. (II.11) is set to zero by requiring that there be no source term for the operator $\langle \mathcal{O} \rangle$ in the field theory action.

Before solving these equations, let us consider the non-superconducting state with $w = 0$. Then the solution of ϕ is $\phi = \mu(1 - z)$. The equation for A_y^3 becomes

$$(-1 + z^3) \frac{d^2 A_y^3(z)}{dz^2} + 3z^2 \frac{dA_y^3(z)}{dz} = 0. \quad (\text{II.14})$$

The solution of this equation takes the following form

$$A_y^3(z) = c_1 + c_2 \left(-\frac{\arctan(\frac{1+2z}{\sqrt{3}})}{\sqrt{3}} + \frac{1}{3} \log(z-1) - \frac{1}{6} \log(1+z+z^2) \right), \quad (\text{II.15})$$

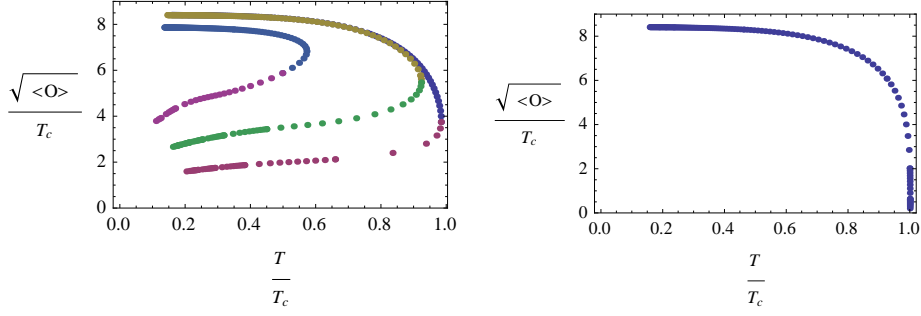


FIG. 1. Plot of the order parameter versus the temperature for different values of the current. The left panel shows three different plots of $\langle \mathcal{O} \rangle$ versus T for $j_y=1/10$, $1/100$, $1/1000$ (from the right to the left). The right panel shows the plot of $\langle \mathcal{O} \rangle$ versus T with zero current, it is clear that the phase transition is a second order transition now.

where c_1 and c_2 are two constants. c_2 must equal to zero since we require that the energy density near the horizon be finite. If c_2 is non-vanishing, then near the horizon the term with $\log(z-1)$ gives a contribution to $F_{z,y}^3 \sim \partial_z A_y^3 \sim c_2 \frac{1}{z-1}$. Then the energy density near the horizon has a contribution from $g^{zz}g^{yy}F_{zy}^2$ which diverges as $1/(z-1)$ at the horizon $z_h = 1$. This unphysical behaviors means that there is no supercurrent for the non-superconducting state since A_y^3 must be a constant. However, the non-superconducting state can have a superfluidity with value c_1 . We will come back to this issue again when we calculate the free energy later.

B. Order Parameter via Temperature

To study the behaviors of a p -wave model with supercurrent means that we have to solve the equations of motion with fixed j_y . We can also solve the equations with a fixed v_y , which corresponds to studying the phases of a superconductor with a fixed superfluidity velocity. This has been done in Ref. [27]. The first important problem to study is how the order parameter changes with the temperature for this holographic superconductor with current.

From Fig. 1 it can be seen that when the supercurrent is not zero, there are two solutions of the order parameter corresponding to a fixed temperature. In the next section we will show that the solution with lower value of the order parameter takes a larger free energy than the solution with larger values of the order parameter and therefore it is unfavorable. The critical temperature decreases as the current increases, which indicates that there should exist a critical current above which there is no superconductivity. When one lowers the temperature from a temperature above the critical one, the order of phase transition at the critical temperature for a fixed current should be a first order one, since the order parameter jumps from zero to a finite value at the critical temperature. Such a jump will certainly change the energy and so requires some latent heat, which implies that the phase transition should be a first order one. This conclusion is the same as the one we shall give from observing the curve of the current j_y versus the superfluid velocity v_y at a fixed temperature. This result is different from the one obtained for the case of a fixed superfluidity velocity, where at small velocity the phase transition is still a second order one. When the velocity becomes larger, the phase transition turns to be of first order [27]. For the s -wave holographic superconductor with current, the order parameter is also bivaluated, and the states with lower value of the condensate have a larger free energy than their counterparts with larger values of the condensate at the same temperature. This is the same as the result of a p -wave holographic superconductor.

C. The Free Energy

To confirm that compared with the state with larger parameter, the state with lower parameter is unfavorable, we need to compare their free energies. The free energy of the field theory is determined by the value of the Yang-Mills action (ignoring the back-reaction of the gauge fields on the metric)

$$S_{\text{YM}} = \int d^4x \mathcal{L}_{\text{YM}} \quad (\text{II.16})$$

evaluated on-shell up to boundary counterterms, $F = -TS_{\text{os}} + \dots$, where the ellipsis denotes boundary terms that we should introduce to regulate the action when needed. The on-shell Yang-Mills action S_{os} is determined by plugging the equations of motion, Eq. (II.8), Eq. (II.9) and Eq. (II.10) into the explicit form of the Yang-Mills Lagrangian (omitting the irrelevant factor $1/4g_{YM}^2$)

$$S_{\text{os}} = \int d^3x (-\phi\phi' + 2z^2 fww' - A_y^3(A_y^3)')|_{z=\epsilon} - \int d^3x \int_{\epsilon}^{z_h} dz \left(\frac{\phi^2 w^2}{1-z^3} + w^2 (A_y^3(z))^2 \right), \quad (\text{II.17})$$

where we have used the coordinate $z = 1/r$, and $z = \epsilon = 0^+$ is the boundary of the AdS spacetime.

To regulate S_{os} , it is important to choose an ensemble. By keeping μ fixed, we are working in the grand canonical ensemble without an additional boundary term. Near the boundary $z = \epsilon$, the fields ϕ and w are determined by Eq. (II.11), Eq. (II.12) and Eq. (II.13) and the three terms $-\phi\phi'$, $-A_y^3(A_y^3)'$ and $2z^2 fww'$ in Eq. (II.17) give $\mu\rho$, $v_y j_y$ and $2w_0 w_1$, respectively. We can see that the on-shell action S_{os} is not divergent and no counterterms are needed. Since w_0 is fixed to be zero, for a spatially homogenous system, the free energy density of the field theory takes the following form

$$F/V = -\mu\rho - v_y j_y + \int_{\epsilon}^{z_h} dz \left(\frac{\phi^2 w^2}{1-z^3} + w^2 (A_y^3(z))^2 \right), \quad (\text{II.18})$$

where $V \equiv \int d^3x$.

In Fig. 2 we present the free energy of a fixed current $j_s = 1/100$ for the two branches of solution. It can be clearly seen that the solution with a larger value of the order parameter has a lower free energy.

Of course, the best way to show that the phase transition at the critical temperature is of first order is to compare the free energy between the superconducting state and the non-superconducting state with current. However, this is not possible in this model, since there is no non-superconducting state with a fixed current as is discussed at the end of Section A. Nevertheless, with the fact that the order parameter goes discontinuously at the phase transition point, there should be no problem to conclude

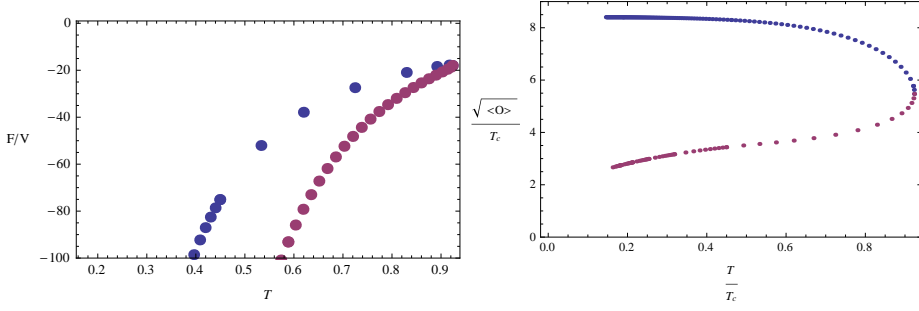


FIG. 2. Plot of the free energy of the superconducting phases for $j_y = 1/100$. The blue dotted line corresponds to the points with a lower value of the condensate at a given temperature. The right panel shows the corresponding plot of the condensation versus T . It can be seen that the lower branch corresponds indeed to states with larger free energy and is thus metastable.

that the phase transition is a first order one.

D. Current via the Superfluidity Velocity

Another physical quantity by which one can compare the difference between the gravity model of superconductor and the G-L theory is the relation between the current and the superfluidity velocity at a fixed temperature. From this relation we can also get the information of the phase transitions at the critical current or critical velocity.

The first two plots in Fig. 3 correspond to the temperature $T/T_c = 0.9836, 0.9229$. For these two temperatures the critical current are $j_c = 1/1000$ and $j_c = 1/100$, respectively. It can be clearly seen that for temperatures close to T_c , where the G-L theory works very well, the plots of j_y versus v_y is the same as that of G-L theory. From these plots we can also know the order of the phase transitions at critical current or critical velocity. For the plots of $T/T_c = 0.9836, 0.9229$, the maximal velocity corresponds to a vanishing current, which means that the phase transitions

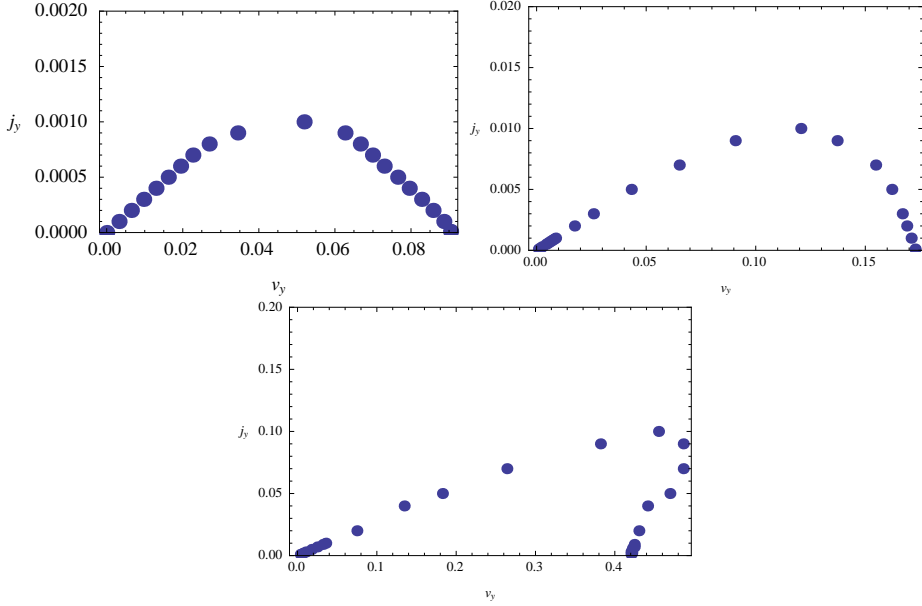


FIG. 3. Plots of the current j_y versus the superfluid velocity v_y at a fixed temperature. The two panels above correspond to $T/T_c = 0.9836, 0.9229$ (from left to right), at which the critical currents j_c are $1/1000$ and $1/100$, respectively. The panel below shows the result for $T/T_c = 0.4836$ corresponding to $j_c = 1/10$.

at critical velocities $j_c = 1/1000$ and $j_c = 1/100$ are of second order.

The third plot corresponding to a larger temperature $T/T_c = 0.4836$ is different. The maximal velocity corresponds to a non-vanishing current, which means that the phase transition at the critical velocity is a first order one. So we can conclude that for small critical velocity the phase transition is a second order transition, while for large enough velocity the phase transition becomes a first order one. These are consistent with the model with a $p + ip$ background at a fixed superfluidity velocity [27]. We can also check our results by the curve of the order parameter versus the temperature for a fixed velocity (rather than a fixed current), which is plotted in Fig 4. From Fig. 4 it can be seen that for a small value of the current $v_y = 0.1$, the condensation goes continuously at the critical temperature, which means that the phase transition is a second order one. While for a larger value of the current $v_y = 0.48$, which corresponds

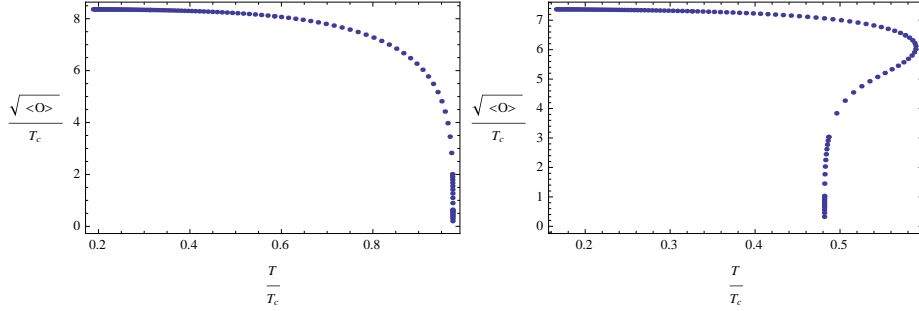


FIG. 4. Plot of the condensation versus the temperature for a fixed velocity. The left panel corresponds to $v_y = 0.1$ and the right panel corresponds to $v_y = 0.48$.

to the maximal value of the current in the third curve of Fig. 3, the condensation goes discontinuously at the critical temperature. This indicates that the phase transition is a first order one. According to the computation of the free energy in Refs. [17, 27], it shows that the curve of the condensation versus the temperature, such as the right one, is indeed the sign of a first order phase transition. An interesting thing is that all the results we obtain above are similar to that of the s -wave model [18, 20].

E. The Critical Current via Temperature

In this subsection we study the critical current j_c for different T near T_c to compare the results with that of the G-L theory. As predicted by G-L theory, j_c is proportional to $(T_c - T)^{3/2}$ when the temperature is close to T_c . As illustrated in Fig. 5, this scaling behavior is indeed obeyed by holographic superconductors for temperatures close to T_c , and this is also the case in the s -wave model [20]. Another prediction of the G-L theory is that, at any fixed temperature, the norm of the condensate decreases monotonically with the velocity from its maximal value, the maximal value $\langle \mathcal{O} \rangle_\infty$ corresponding to zero velocity and zero current. As is shown in Fig. 3, the critical current is reached before the velocity reaches its maximal value. The norm of the condensate has an intermediate value $\langle \mathcal{O} \rangle_c$ at the maximal current. The G-L theory

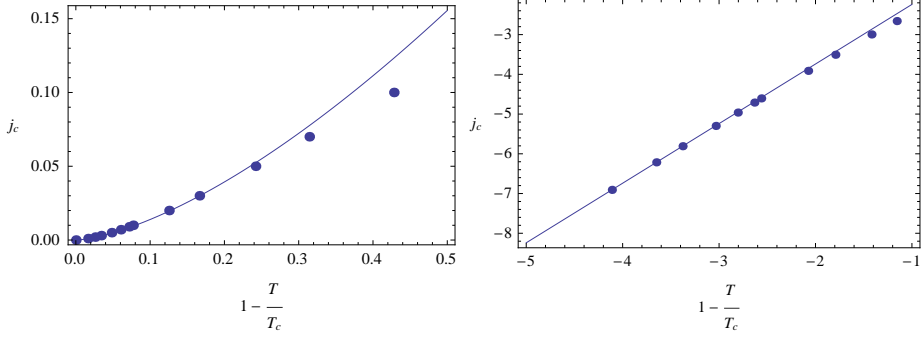


FIG. 5. Plot of the critical current versus the temperature. The right panel shows a log-log plot from which we can read off the critical exponent, getting 1.497, which agrees with the expected GL scaling of $3/2$ within numerical precision. The left panel shows the j_c versus $(1 - T/T_c)$, the solid line is $0.44(1 - T/T_c)^{3/2}$.

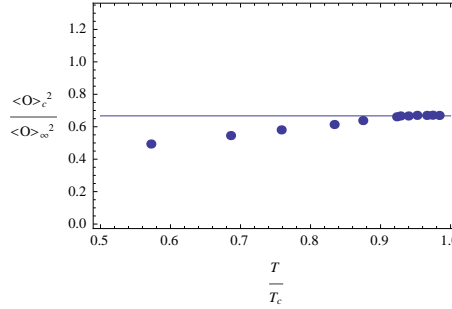


FIG. 6. Plot of the ratio $(\langle \mathcal{O} \rangle_c / \langle \mathcal{O} \rangle_\infty)^2$ versus the temperature. The solid line corresponds to the value of $2/3$ predicted by the G-L theory and it also appears in the s -wave model.

tells us that the squared ratio of $\langle \mathcal{O} \rangle_c$ to the maximal condensation $\langle \mathcal{O} \rangle_\infty$ is exactly equal to $2/3$. From Fig. 6 it can be seen that this is indeed the same case for the p -wave holographic superconductor.

III. THE $p + ip$ BACKGROUNDS WITH CURRENT

The EYM theory with a $p + ip$ background has the following ansatz [22, 23],

$$A = \phi(z)T^3 dt + A_y^3(z)T^3 dy + w(z)T^1 dx + w(z)T^2 dy. \quad (\text{III.19})$$

With this ansatz, the EOMs are

$$(-1+z^3)^2 \frac{d^2 w(z)}{dz^2} + 3z^2(-1+z^3) \frac{dw(z)}{dz} + (-1+z^3)w^3(z) + \phi^2(z)w(z) + \frac{(-1+z^3)(A_y^3(z))^2 w(z)}{2} = 0, \quad (\text{III.20})$$

$$(-1+z^3) \frac{d^2 \phi''(z)}{dz^2} + 2\phi(z)w^2(z) = 0, \quad (\text{III.21})$$

and

$$(-1+z^3) \frac{d^2 A_y^3(z)}{dz^2} + 3z^2 \frac{dA_y^3(z)}{dz} + A_y^3(z)w^2(z) = 0. \quad (\text{III.22})$$

The boundary conditions for the fields w , ϕ and A_y^3 are the same as those in the case of p -wave background discussed in Section II.A.

After solving the EOMs numerically, we find similar results as those of the p -wave holographic superconductor discussed in last section. These results are shown in Figs. 7-9.

Until now, for both p wave and $p+ip$ wave backgrounds of non-Abelian holographic superconductor with DC current along the y direction, we get results extremely close to those of the s -wave model. A first look of the EOMs of both p wave and $p+ip$ wave holographic superconductors with DC current makes us find that the Eq. (II. 8), Eq. (II. 9), Eq. (II. 10), Eq. (III. 20), Eq. (III. 21) and Eq. (III. 22) are different from the corresponding ones of the s -wave model in Ref. [18]. For the non-Abelian holographic superconductors the expansion of the field $w(z)$ goes as a constant plus a term linear in z (equation II.11). In Ref. [18], their scalar field $\psi(z)$'s expansion goes as a linear term plus a term quadratic in z (equation (10)). If we do a field redefinition, such that $\psi(z) = zw(z)/\sqrt{2}$ from the scalar field of Ref. [18], Eq. (II.8) for $w(z)$ and the corresponding one in Ref. [18] will differ by a single term proportional to

$$z(-1+z^3)w(z). \quad (\text{III.23})$$

But the equations for the scalar potential and the vector potential are the same as the corresponding ones in Ref. [18]. According to our computation, this term

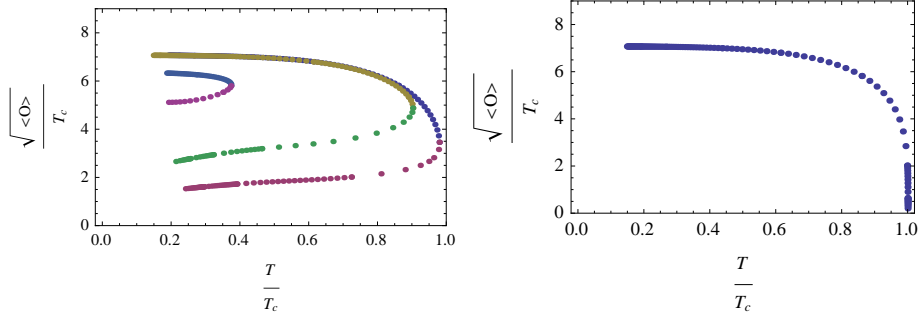


FIG. 7. Plot of the order parameter versus the temperature for different values of current for the case of $p + ip$ background. The left panel shows the three different plots of $\langle \mathcal{O} \rangle$ versus the temperature for $j_y = 1/10, 1/100, 1/1000$ (from the right to the left). The right panel shows the plot of $\langle \mathcal{O} \rangle$ versus the temperature with zero current.

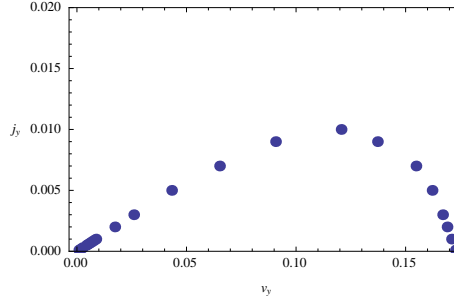


FIG. 8. Plot of the current j_y versus the superfluid velocity v_y for $p + ip$ background at a fixed temperature $T/T_c = 0.9284$, at which the critical current is $1/100$.

(Eq. (III.23)) will not affect the qualitative results but will change the quantitative results. For the $p + ip$ wave model, the situation is different due to the w^3 term in Eq. (III.20). In the next section we will turn on j_x rather than j_y for both p wave and $p + ip$ wave backgrounds, and we will see that these two backgrounds are different with a nonvanishing A_x .

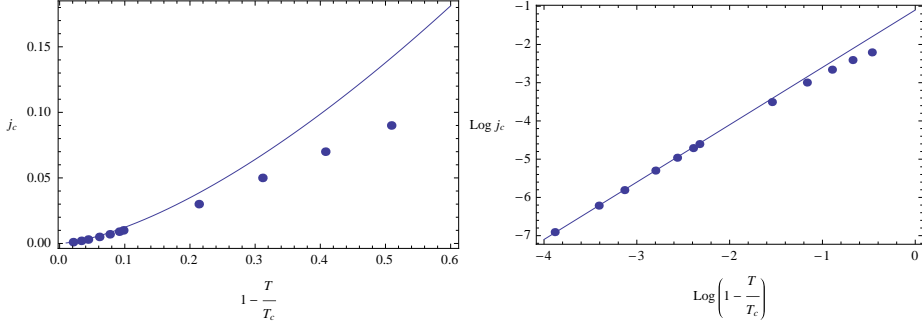


FIG. 9. Plot of the critical current versus the temperature for $p + ip$ background. The right panel shows a log-log plot from which we can read off the critical exponent, getting 1.498, which agrees with the expected G-L scaling of $3/2$ within numerical precision. The left panel shows the j_c versus $(1 - T/T_c)$, the solid line is $0.39(1 - T/T_c)^{3/2}$.

IV. CURRENTS ALONG THE x DIRECTION

Now let us study whether there is any difference between the p -wave, $p + ip$ and s -wave model with DC current. For the $p + ip$ wave background, by assuming a nonvanishing A_x^3 , the EOMs become

$$\begin{aligned} & (-1 + z^3)^2 \frac{d^2 w(z)}{dz^2} + 3z^2(-1 + z^3) \frac{dw(z)}{dz} \\ & + (-1 + z^3)w^3(z) + \phi^2(z)w(z) + \frac{(-1 + z^3)(A_x^3(z))^2 w(z)}{2} = 0, \end{aligned} \quad (\text{IV.24})$$

$$(-1 + z^3) \frac{d^2 \phi''(z)}{dz^2} + 2\phi(z)w^2(z) = 0, \quad (\text{IV.25})$$

and

$$(-1 + z^3) \frac{d^2 A_x^3(z)}{dz^2} + 3z^2 \frac{dA_x^3(z)}{dz} + A_x^3(z)w^2(z) = 0. \quad (\text{IV.26})$$

We can see that these equations are the same as Eq. (III. 20), Eq. (III. 21) and Eq. (III. 22). Therefore, the same results for j_x will be expected. We can also turn on a current along an arbitrary direction, which can be done by turning on both j_x and j_y with a relationship like $j_x = k j_y$. For example, a current along the direction of 45 degree to the x direction corresponds to $k = 1$. With both j_x and j_y , the EOMs

becomes

$$(-1+z^3)^2 \frac{d^2 w(z)}{dz^2} + 3z^2(-1+z^3) \frac{dw(z)}{dz} + (-1+z^3)w^3(z) + \phi^2(z)w(z) + \frac{(-1+z^3)((A_x^3(z))^2 + (A_y^3(z))^2)w(z)}{2} = 0, \quad (\text{IV.27})$$

$$(-1+z^3) \frac{d^2 \phi''(z)}{dz^2} + 2\phi(z)w^2(z) = 0, \quad (\text{IV.28})$$

$$(-1+z^3) \frac{d^2 A_x^3(z)}{dz^2} + 3z^2 \frac{dA_x^3(z)}{dz} + A_x^3(z)w^2(z) = 0. \quad (\text{IV.29})$$

and

$$(-1+z^3) \frac{d^2 A_y^3(z)}{dz^2} + 3z^2 \frac{dA_y^3(z)}{dz} + A_y^3(z)w^2(z) = 0. \quad (\text{IV.30})$$

Since the equations for A_x and A_y are the same, $j_x = kj_y$ means that $A_x = kA_y$. Then the EOMs reduce to

$$(-1+z^3)^2 \frac{d^2 w(z)}{dz^2} + 3z^2(-1+z^3) \frac{dw(z)}{dz} + (-1+z^3)w^3(z) + \phi^2(z)w(z) + \frac{(-1+z^3)(k^2+1)(A_y^3(z))^2 w(z)}{2} = 0, \quad (\text{IV.31})$$

$$(-1+z^3) \frac{d^2 \phi''(z)}{dz^2} + 2\phi(z)w^2(z) = 0, \quad (\text{IV.32})$$

and

$$(-1+z^3) \frac{d^2 A_y^3(z)}{dz^2} + 3z^2 \frac{dA_y^3(z)}{dz} + A_y^3(z)w^2(z) = 0. \quad (\text{IV.33})$$

Now the value of current $j = j_y \sqrt{k^2+1}$. After a redefinition $A_y \rightarrow A_y / \sqrt{k^2+1}$, the EOMs turn back to Eq. (III. 20), Eq. (III. 21) and Eq. (III. 22) with the same magnitude of DC current. From this fact we conclude that if a current in both the x and y direction is present, the equations of motion only depend on the magnitude of the current and not on its direction. Then the supercurrent in $p + ip$ holographic superconductor is isotropic, which is similar to the case of the s wave model.

For the p wave background with j_x , the EOMs are

$$(-1+z^3)^2 \frac{d^2 w(z)}{dz^2} + 3z^2(-1+z^3) \frac{dw(z)}{dz} + \phi^2(z)w(z) = 0, \quad (\text{IV.34})$$

$$(-1 + z^3) \frac{d^2 \phi''(z)}{dz^2} + \phi(z) w^2(z) = 0, \quad (\text{IV.35})$$

and

$$(-1 + z^3) \frac{d^2 A_x^3(z)}{dz^2} + 3z^2 \frac{dA_x^3(z)}{dz} = 0. \quad (\text{IV.36})$$

From these equations it can be seen that A_x is independent of w . A_x is totally determined by Eq. (IV.29). This equation is also the same as Eq. (II.14). According to the study of Eq. (II.14) in Section II. A, A_x must be a constant. From this we conclude that there is no supercurrent along the x direction. Just as we have discussed for the $p + ip$ background, we can also try to turn on a current along an arbitrary direction, which means that we have to turn on both A_x^3 and A_y^3 . Then the EOMs turn to

$$(-1 + z^3)^2 \frac{d^2 w(z)}{dz^2} + 3z^2(-1 + z^3) \frac{dw(z)}{dz} + \phi^2(z) w(z) + (-1 + z^3)(A_y^3(z))^2 w(z) = 0, \quad (\text{IV.37})$$

$$(-1 + z^3) \frac{d^2 \phi''(z)}{dz^2} + \phi(z) w^2(z) = 0, \quad (\text{IV.38})$$

$$(-1 + z^3) \frac{d^2 A_y^3(z)}{dz^2} + 3z^2 \frac{dA_y^3(z)}{dz} + A_y^3(z) w^2(z) = 0. \quad (\text{IV.39})$$

and

$$(-1 + z^3) \frac{d^2 A_x^3(z)}{dz^2} + 3z^2 \frac{dA_x^3(z)}{dz} = 0. \quad (\text{IV.40})$$

It is also clear that A_x is still independent of w , so there also should be no j_x , as has been discussed. The current can only flow along the y direction, then the DC current in the p wave holographic superconductor is anisotropic.

V. DISCUSSION AND CONCLUSION

In this paper, we study the p -wave and the $p + ip$ wave holographic superconductor with DC supercurrent. For the p wave background with DC current along the y direction and the $p + ip$ wave background with DC current along both x and y directions,

the results near the critical temperature agree quantitatively with several properties of the Ginzburg-Landau theory. For example, the squared ratio of the maximal condensate to the minimal condensate is equal to two thirds, the critical current is proportional to $(T_c - T)^{3/2}$. While for the p wave model there is no supercurrent along the x direction. However, it is interesting to note that the non-Abelian holographic superconductors show the same mean-field behaviors as the s -wave model, which are also the results of the G-L theory. These results make us believe that the holographic description of superconductors indeed contains some physics of real world superconductors. Maybe an analytical analysis of these two models are helpful to explaining why the holographic models of s -wave superconductor and p -wave superconductor show similar results [36, 37].

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- [1] J. M. Maldacena, “The large N limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. **2** (1998) 231 [arXiv:hep-th/9711200].
 - [2] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “Gauge Theory Correlators from Non-Critical String Theory,” Phys. Lett. B **428**, 105 (1998) [arXiv:hep-th/9802109].
 - [3] E. Witten, “Anti-de Sitter space and holography,” Adv. Theor. Math. Phys. **2**, 253 (1998) [arXiv:hep-th/9802150].

- [4] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri, and Y. Oz, “Large N Field Theories, String Theory and Gravity,” Phys. Rept. **323**, 183 (2000) [arXiv:hep-th/9905111].
- [5] S. S. Gubser, “Breaking an Abelian gauge symmetry near a black hole horizon,” Phys. Rev. D **78**, 065034 (2008).
- [6] C. P. Herzog, “Lectures on Holographic Superfluidity and Superconductivity,” J. Phys. A **42** (2009) 343001 [arXiv:0904.1975 [hep-th]].
- [7] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, “Building a Holographic Superconductor,” Phys. Rev. Lett. **101**, 031601 (2008) [arXiv:0803.3295 [hep-th]].
- [8] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, “Holographic Superconductors,” JHEP **0812**, 015 (2008) [arXiv:0810.1563 [hep-th]].
- [9] G. T. Horowitz and M. M. Roberts, “Holographic Superconductors with Various Condensates,” Phys. Rev. D **78** 126008 (2008) [arXiv:0810.1077 [hep-th]].
- [10] K. Maeda and T. Okamura, “Characteristic length of an AdS/CFT superconductor,” Phys. Rev. D **78**, 106006 (2008) [arXiv:0809.3079 [hep-th]].
- [11] Oscar J.C. Dias, Ricardo Monteiro, Harvey S. Reall, Jorge E. Santos, “A scalar field condensation instability of rotating anti-de Sitter black holes,” JHEP **1011**, 036, (2010).
- [12] Jiunn-Wei Chen, Ying-Jer Kao, Debaprasad Maity, Wen-Yu Wen, Chen-Pin Yeh, “Towards A Holographic Model of D-Wave Superconductors ,” Phys. Rev. D **81**:106008, 2010.
- [13] Francesco Benini, Christopher P. Herzog, and Amos Yarom, “Holographic Fermi arcs and a d-wave gap,” [arXiv:1006.0731v1].
- [14] Francesco Benini, Christopher P. Herzog, Rakibur Rahman, Amos Yarom, “Gauge gravity duality for d-wave superconductors: prospects and challenges,” [arXiv:1007.1981].
- [15] H. Zeng, Z. Fan, H. S. Zong, “Characteristic length of a Holographic Superconductor with *d*-wave gap,” Phys. Rev. D **82**, 126014 (2010) [arXiv:1006.5483 [hep-th]].

- [16] H. Zeng, Z. Fan, H. S. Zong, “d-wave Holographic Superconductor Vortex Lattice and Non-Abelian Holographic Superconductor Droplet,” *Phys. Rev. D* **82**, 126008 (2010)[arXiv:1007.4151 [hep-th]].
- [17] C. P. Herzog, P. K. Kovtun and D. T. Son, “Holographic model of superfluidity,” *Phys. Rev. D* **79**, 066002 (2009) [arXiv:0809.4870 [hep-th]].
- [18] Pallab Basu, Anindya Mukherjee, Hsien-Hang Shieh, “Supercurrent: Vector Hair for an AdS Black Hole,” *Phys.Rev.D*79:045010,(2009) [arXiv:0809.4494 [hep-th]].
- [19] Daniel Arean, Pallab Basu, Chethan Krishnan, “The Many Phases of Holographic Superfluids,” *JHEP* 10 (2010) 006.
- [20] Daniel Arean, Matteo Bertolini, Jarah Evslin, Tomas Prochazka, “On Holographic Superconductors with DC Current,” [arXiv:1003.5661]
- [21] S. S. Gubser, “Colorful horizons with charge in anti-de Sitter space,” *Phys. Rev. Lett.* **101**, 191601 (2008).
- [22] S. S. Gubser and S. S. Pufu, “The gravity dual of a p-wave superconductor,” *JHEP* **0811**, 033 (2008) [arXiv:0805.2960 [hep-th]].
- [23] M. M. Roberts and S. A. Hartnoll, “Pseudogap and time reversal breaking in a holographic superconductor,” *JHEP* **0808**, 035 (2008) [arXiv:0805.3898 [hep-th]].
- [24] M. Ammon *et al.*, On Holographic p-wave Superfluids with Back-reaction. *Phys. Lett. B* **686**, 192 (2010) [arXiv:0912.3515 [hep-th]].
- [25] Yves Brihaye, Betti Hartmann, “Holographic Superconductors in 3+1 dimensions away from the probe limit,” arXiv:1003.5130.
- [26] P. Basu *et al.*, Hard-gapped Holographic Superconductors. *Phys. Lett. B* 689,45 (2010) arXiv:0911.4999 [hep-th].
- [27] Pallab Basu, Jianyang He, Anindya Mukherjee, Hsien-Hang Shieh, “Superconductivity from D3/D7: Holographic Pion Superfluid,” *JHEP* 0911:070,(2009) [arXiv:0810.3970 [hep-th]].

- [28] C. P. Herzog and S. S. Pufu, “The Second Sound of $SU(2)$,” JHEP 04 (2009) 126 [arXiv:0902.0409 [hep-th]].
- [29] M. Ammon, J. Erdmenger, M. Kaminski and P. Kerner, “Superconductivity from gauge/gravity duality with flavor,” Phys. Lett. **B680** (2009) [arXiv:0810.2316 [hep-th]].
- [30] M. Ammon, J. Erdmenger, M. Kaminski and P. Kerner, “Flavor Superconductivity from Gauge/Gravity Duality,” JHEP 10 (2009) 067 [arXiv:0903.1864 [hep-th]].
- [31] K. Peeters, J. Powell, M. Zamaklar, “Exploring colourful holographic superconductors,” JHEP **0909**, 101, (2009) [arXiv:0907.1508 [hep-th]].
- [32] S. S. Gubser, F. D. Rocha, A. Yarom, “Fermion correlators in non-abelian holographic superconductors,” [arXiv:1002.4416]
- [33] H. Zeng, Z. Fan, Z. Ren, “Time Reversal Symmetry Breaking Holographic Superconductor in Constant External Magnetic Field,” Phys. Rev. D **80**, 066001 (2009) [arXiv:0906.2323 [hep-th]].
- [34] H. Zeng, Z. Fan, H. S. Zong, “Superconducting Coherence Length and Magnetic Penetration Depth of a p-wave Holographic Superconductor,” Phys. Rev. D **81**, 106001 (2010) [arXiv:0912.4928 [hep-th]].
- [35] M. Kaminski, “Flavor Superconductivity and Superfluidity,” [arXiv:1002.4886]
- [36] G. Siopsis, J. Therrien, “Analytic calculation of properties of holographic superconductors,” [arXiv:1003.4275].
- [37] Hua-Bi Zeng, Xin Gao, Yu Jiang, Hong-Shi Zong, “Analytical Computation of Critical Exponents in Several Holographic Superconductors,” arXiv:1012.5564.